Latin Square Design

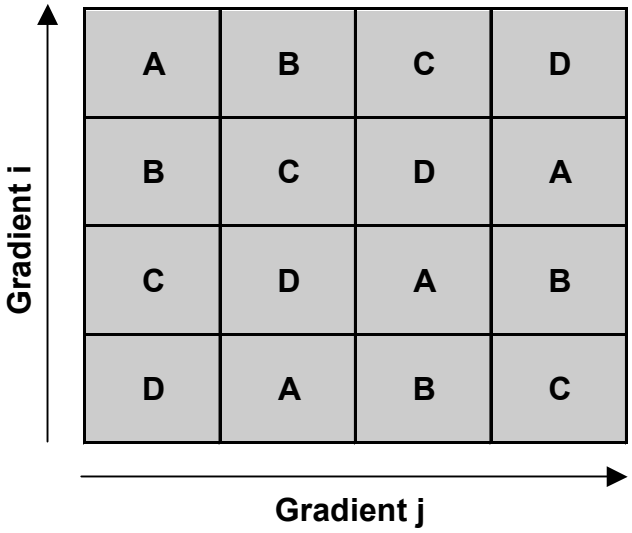
Layout

Column

Row	1	2	3	4
1	A	B	C	D
2	B	C	D	A
3	C	D	A	B
4	D	A	B	C

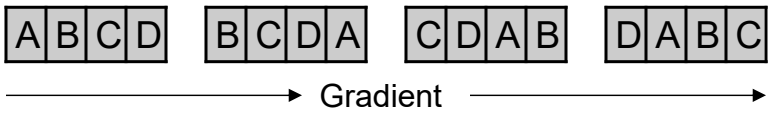
Latin Square Design

Latin Square - Row / Column Layout



Latin Square Design
Linear Layout

Latin Square - Row / Column Layout



Latin Square Design
Linear Additive Model

$$Y_{ijk} = \mu + R_i + \delta_{(i)} + C_j + \omega_{(j)} + T_k + \varepsilon_{(ijk)}$$

Where:

Y_{ijk} = variable to be analyzed from i^{th} row and j^{th} column and the k^{th} treatment
μ = overall mean
R_i = effect of the i^{th} row
$\delta_{(i)}$ = restriction error associated with rows
C_j = effect of the j^{th} column
$\omega_{(j)}$ = restriction error associated with columns
T_k = effect of the k^{th} treatment
$\varepsilon_{(ijk)}$ = residual error

Latin Square Design

Expected Mean Squares

Source	df	EMS
R_i	t - 1	
C_j	t - 1	
T_k	t - 1	$\sigma^2 + t\Phi(T)$
$\varepsilon_{(ijk)}$	(t - 1)(t - 2)	σ^2

Latin Square Design

Example: Alfalfa Inoculum Study

(Petersen, 1994)

Treatments:

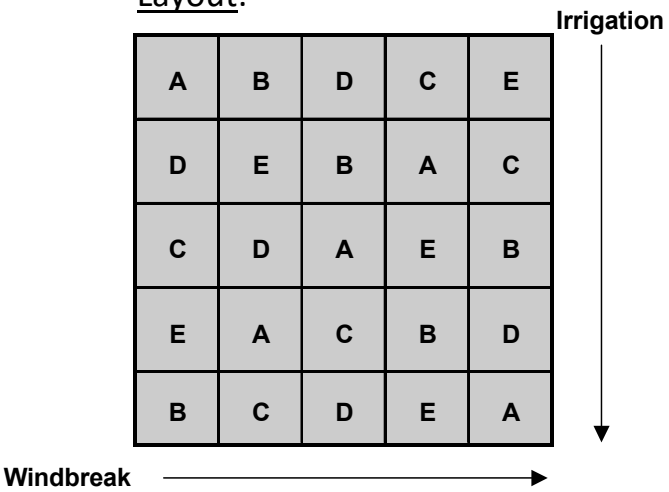
Rows - distance from irrigation source

Columns - distance from windbreak

Inoculum - A, B, C, D, E = Control

Example: Alfalfa Inoculum Study

Layout:



Example: Alfalfa Inoculum Study

ANOVA

Source	df	SS	MS	F
Rows	4	87.40	21.85	
Columns	4	16.56	4.14	
Treatments	4	155.90	38.97	12.69 **
Error	12	36.80	3.07	

Example: Alfalfa Inoculum Study

Standard Errors:

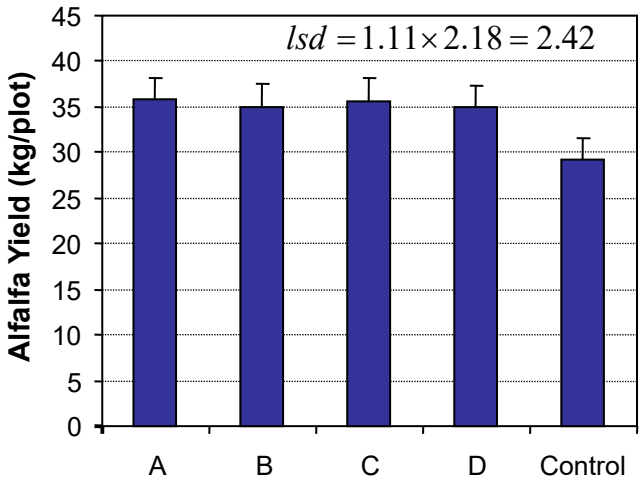
Standard error of the mean

$$S_{\bar{Y}} = \sqrt{\frac{MS_E}{t}} = \sqrt{\frac{3.07}{5}} = 0.78$$

Standard error of the mean difference

$$S_{\bar{d}} = \sqrt{\frac{2MS_E}{t}} = \sqrt{\frac{2(3.07)}{5}} = 1.11$$

Example: Alfalfa Inoculum Study



Latin Square Design

Relative Efficiency

$$RE_{col} = \frac{MS_{RCBD}}{MS_{LS}}$$

$$MS_{RCBD} = \frac{MS_{col} + (t-1)MS_{error}}{t}$$

Example: Alfalfa Inoculum Study

Relative Efficiency

$$RE_{col} = \frac{MS_{RCBD}}{MS_{LS}} = \frac{3.28}{3.07} = 1.08$$

$$MS_{RCBD} = \frac{MS_{col} + (t-1)MS_{error}}{t} = \frac{4.14 + 4(3.07)}{5} = 3.28$$

Latin Square Design

Rows and Columns ≠ Treatments

Biochar Example

Treatment (tons biochar/acre)

	Col: 1		Col: 2		Col: 3	
Row: 1	0	30	20	10	40	50
Row: 2	40	50	30	0	20	10
Row: 3	20	10	40	50	0	30

Latin Square Design

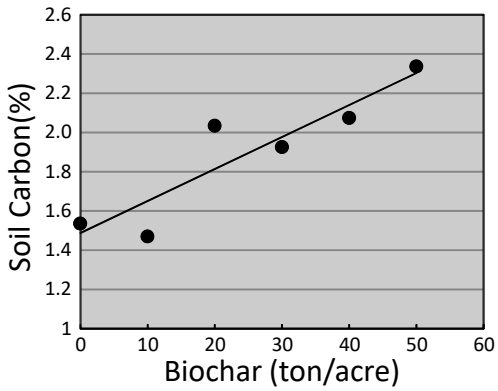
Rows and Columns ≠ Treatments

ANOVA: Soil Carbon

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Row	2	0.43064379	0.21532190		
Col	2	0.10664786	0.05332393		
Trt	5	1.67075494	0.33415099	3.99	0.0410
linear	1	1.39608059	1.39608059	16.68	0.0035
quad	1	0.00001384	0.00001384	0.00	0.9901
Error	8	0.66957726	0.08369716		

Latin Square Design

Rows and Columns ≠ Treatments



Latin Square Design

Growth Chamber Experiment

Treatments:

Chamber	4
Period	4
Temperature	4 - A B C D

Latin Square Design

Growth Chamber Experiment

Chamber \ Period	1	2	3	4
1	A	B	C	D
2	B	C	D	A
3	C	D	A	B
4	D	A	B	C

Latin Square Design

Growth Chamber Experiment

$$Y_{ijk} = \mu + P_i + \delta_{(i)} + C_j + \omega_{(j)} + T_k + \varepsilon_{(ijk)}$$

Source	df	EMS
P_i	p - 1	
C_j	c - 1	
T_k	t - 1	$\sigma^2 + t\Phi(T)$
$\varepsilon_{(ijk)}$	(t - 1)(t - 2)	σ^2

Latin Square Design
Multiple Squares

Layout:

Square 1

A	B	C
B	C	A
C	A	B

Square 2

B	C	A
A	B	C
C	A	B

Latin Square Design
Multiple Squares

Linear Additive Model:

$$Y_{ijkl} = \mu + S_i + R_{(i)j} + \delta_{(ij)} + C_{(i)k} + \omega_{(ik)} + T_l + \varepsilon_{(ijkl)}$$

Where:

Y_{ijk} = variable to be analyzed from i^{th} square, j^{th} row, k^{th} column and l^{th} treatment

μ = overall mean

S_i = effect of the i^{th} square

$R_{(i)j}$ = effect of the j^{th} row within i^{th} square

$\delta_{(ij)}$ = restriction error associated with rows

$C_{(i)k}$ = effect of the k^{th} column within i^{th} square

$\omega_{(ik)}$ = restriction error associated with columns

T_l = effect of the l^{th} treatment

$\varepsilon_{(ijkl)}$ = residual error

Latin Square Design

Multiple Squares

Expected Mean Squares:

Source	df	EMS
S_i	s - 1	
$R_{(i)j}$	s(t - 1)	
$C_{(i)k}$	s(t - 1)	
T_l	t - 1	$\sigma^2 + st\Phi(T)$
$\varepsilon_{(ijkl)}$	(st - s -1)(t - 1)	σ^2

Latin Rectangle

Layout

Square 1			Square 2		
A	**B**	**C**	**B**	**C**	**A**
B	**C**	**A**	**A**	**B**	**C**
C	**A**	**B**	**C**	**A**	**B**

Latin Rectangle

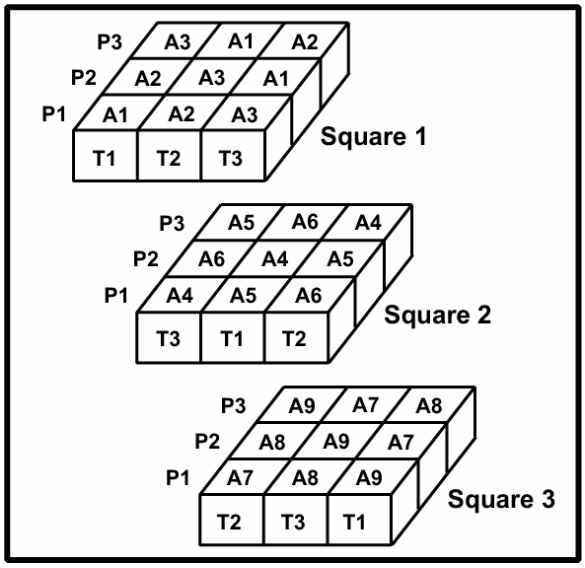
ANOVA

$$Y_{ijkl} = \mu + R_i + \delta_{(i)} + C_j + \omega_{(j)} + T_k + \varepsilon_{(ijk)}$$

Source	df	EMS
R_i	t - 1	
C_j	st - 1	
T_k	t - 1	$\sigma^2 + st\Phi(T)$
$\varepsilon_{(ijk)}$	(st - 2)(t - 1)	σ^2

Multiple Squares

Example: Pasture Experiment



Multiple Squares

Example: Pasture Experiment ANOVA

Source	df	EMS	F-Test
Square (S)	(s - 1)	$\sigma^2 + st\sigma^2_{A(S)} + t^2\sigma^2_S$	
Animal (A) / Square	s(a - 1)	$\sigma^2 + st\sigma^2_{A(S)}$	
Period (P)	(p - 1)	$\sigma^2 + t\sigma^2_{SP} + st\Phi P$	MS_P / MS_{SP}
Square x Period	(s - 1)(p - 1)	$\sigma^2 + t\sigma^2_{SP}$	MS_{SP} / MS_{RES}
Treatment (T)	(t - 1)	$\sigma^2 + t\sigma^2_{ST} + st\Phi T$	MS_T / MS_{ST}
Square x Treatment	(s - 1)(t - 1)	$\sigma^2 + t\sigma^2_{ST}$	MS_{ST} / MS_{RES}
Residual / Square	s(p - 1)(t - 2)	σ^2	

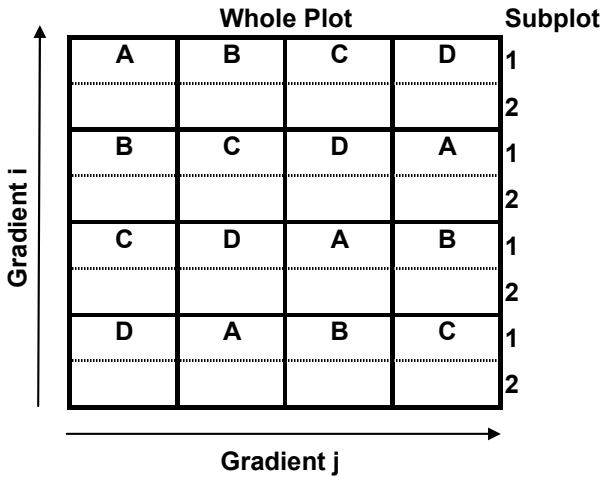
Example: Pasture Experiment

Crossover ANOVA

Source	df	EMS	F-Test
Square (S)	(s - 1)	$\sigma^2 + t^2\sigma^2_S$	
Animal (A) / Square	s(a - 1)	$\sigma^2 + t^2\sigma^2_{A(S)}$	
Period (P)	(p - 1)	$\sigma^2 + t^2\Phi P$	MS_P / MS_{RES}
Treatment (T)	(t - 1)	$\sigma^2 + t^2\Phi T$	MS_T / MS_{RES}
Residual / Square	(p - 1)(st - 2)	σ^2	

Latin Square Split Plot

Layout



Latin Square Split Plot

Linear Additive Model

$$Y_{ijkl} = \mu + R_i + C_j + W_k + RCW_{(ijk)} + S_l + RS_{ik} + CS_{jk} + WS_{kl} + \varepsilon_{(ijkl)}$$

Where:

Y_{ijkl} =	variable to be analyzed from i^{th} row and j^{th} column and the k^{th} and l^{th} treatments
μ =	overall mean
R_i =	effect of the i^{th} row
C_j =	effect of the j^{th} column
W_k =	effect of the k^{th} main treatment
$RCW_{(ijk)}$ =	residual error associated with main-plot treatment (Error a)
S_l =	effect of the l^{th} sub treatment
RS_{il} =	interaction effect between i^{th} row and l^{th} sub-plot treatment
CS_{jl} =	interaction effect between j^{th} column and l^{th} sub-plot treatment
WS_{kl} =	interaction effect between k^{th} main and l^{th} sub-plot treatment
$\varepsilon_{(ijkl)}$ =	residual error associated with sub plots (Error b)

Latin Square Split Plot

ANOVA

Source	df	MS	F-test
Row	t - 1		
Column	t - 1		
WP Treatment	t - 1	MS_W	MS_W / MS_a
Error a	(t - 1)(t - 2)	MS_a	
SP Treatment	s - 1	MS_S	MS_S / MS_b
R x S	(t - 1)(s - 1)	MS_{RS}	MS_{RS} / MS_b
C x S	(t - 1)(s - 1)	MS_{CS}	MS_{CS} / MS_b
W x S	(t - 1)(s - 1)	MS_{WS}	MS_{WS} / MS_b
Error b	(t - 1)(t - 2)(s -1)	MS_b	

Latin Square Split Plot

Sugar Beet Example

SAS User's Guide, Data from Smith (1951)

Treatments:

Row – 1, 2, 3, 4, 5, 6

Column – 1, 2, 3, 4, 5, 6

Variety – A, B, C, D, E, F (main plot)

Harvest – 1, 2 (sub plot)

Latin Square Split Plot
Sugar Beet Example

SAS Code

```
proc anova;
  class Column Row Variety Harvest;
  model Y=Row Column Variety Row*Column*Variety
             Harvest Harvest*Row Harvest*Column
             Harvest*Variety;
  test h=Variety e=Row*Column*Variety;
  test h=Harvest e=Harvest*Row;
  means variety / lsd e=Row*Column*Variety;
  means harvest / lsd e=Harvest*Row;
run;
```

Latin Square Split Plot
Sugar Beet Example

SAS Output

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	51	100.0054	1.960891	6.17	<.0001
Error	20	6.357778	0.317889		
Corrected Total	71	106.3632			

R-Square	Coeff Var	Root MSE	Y Mean
0.940226	3.187154	0.563816	17.69028

Latin Square Split Plot

Sugar Beet Example

SAS Output

Source	DF	Anova SS	Mean Square	F Value	Pr > F
Row	5	4.320694	0.864139	2.72	0.0495
Column	5	1.574028	0.314806	0.99	0.4483
Variety	5	20.61903	4.123806	12.97	<.0001
Column*Row* Variety	20	3.254444	0.162722	0.51	0.9286
Harvest	1	60.68347	60.68347	190.9	<.0001
Row*Harvest	5	7.717361	1.543472	4.86	0.0045
Column* Harvest	5	1.090694	0.218139	0.69	0.6394
Variety* Harvest	5	0.745694	0.149139	0.47	0.7947

Latin Square Split Plot

Sugar Beet Example

SAS Output

Tests of Hypotheses Using the Anova MS for Column*Row*Variety as an Error Term

Source	DF	Anova SS	Mean Square	F Value	Pr > F
Variety	5	20.61903	4.123806	25.34	<.0001

Tests of Hypotheses Using the Anova MS for Row*Harvest as an Error Term

Source	DF	Anova SS	Mean Square	F Value	Pr > F
Harvest	1	60.68347	60.68347	39.32	0.0015

Latin Square Split Plot

Sugar Beet Example

SAS Output

```
Alpha                           0.05
Error Degrees of Freedom          20
Error Mean Square           0.162722
Critical Value of t          2.08596
Least Significant Difference  0.3435


t Grouping          Mean      N    Variety
          A       18.8250     12    5
          B       17.8167     12    2
     C    B       17.4750     12    3
     C            17.3667     12    6
     C            17.3667     12    1
     C            17.2917     12    4
```

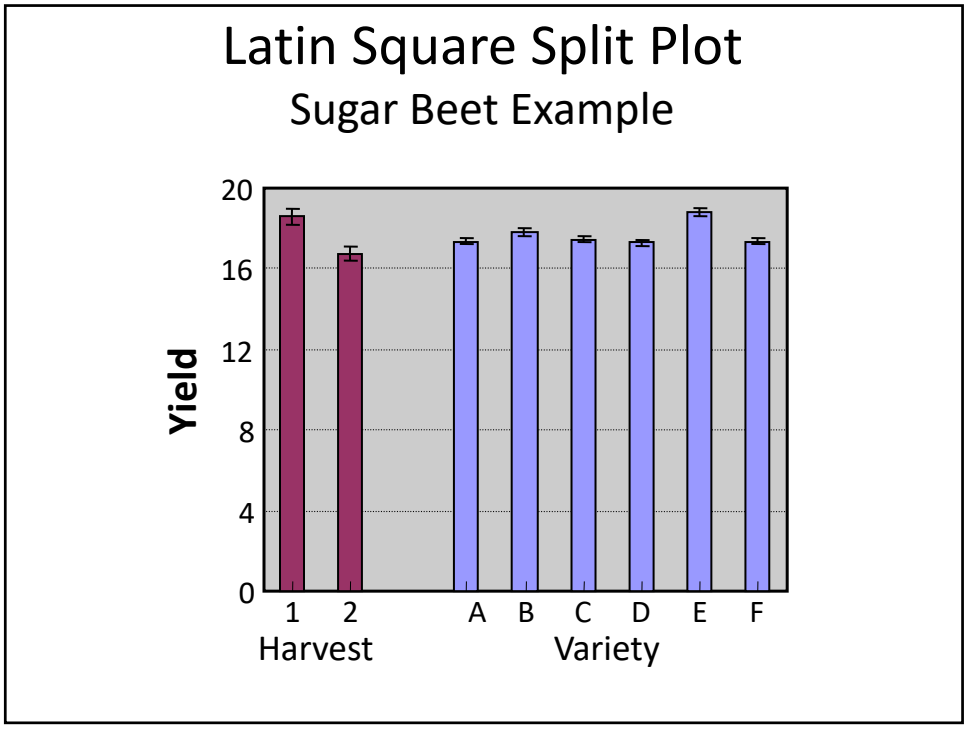
Latin Square Split Plot

Sugar Beet Example

SAS Output

```
Alpha                           0.05
Error Degrees of Freedom           5
Error Mean Square           1.543472
Critical Value of t          2.57058
Least Significant Difference  0.7527


t Grouping          Mean      N    Harvest
          A       18.6083     36    1
          B       16.7722     36    2
```


Latin Square Split Plot
Sugar Beet Example
Yield
20
16
12
8
4
0
1
2
A
B
C
D
E
F
Harvest
Variety